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A COMPARATIVE STUDY OF AMERICAN DIRECT CURRENT WATTHOUR METERS

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I. INTRODUCTION

Although many excellent discussions of the watthour meter have been published, it was thought that a thorough study of all American makes of a given type of meter, made at the same time, would be of considerable interest and value. In order to obtain such data each of the six American manufacturers of direct-current watthour meters was requested by the Bureau of Standards to submit three 5-ampere, 220-volt, 3-wire meters for a

comparative test.¹ In most cases the results given in the following discussion are the average values obtained from tests of three meters.

It should be stated at the outset that this test was not intended to be competitive. No attempt has been made to rank the meters according to the results of the tests.² Any such grading is very likely to be misleading as the various features studied are of unequal importance. Moreover, the characteristics required for different kinds of service are unlike and would be weighted differently by different engineers. It must be noted that while laboratory tests are valuable, they do not form a sufficient basis upon which to pass final judgment on a type of meter. For such a purpose practical experience with meters is necessary, such as is best obtained in central-station practice.

II. PERFORMANCE DATA

1. **Load Curves.**—Before any tests were made the meters were inspected to see that they were in proper running order. They were then adjusted to be as nearly correct as practicable at the 10 per cent and 100 per cent loads.

In this discussion the load curve of the watthour meter is the plot of the rate against load, the rate being defined as the ratio of the watthours recorded by the meter to the actual watthours. Such a curve has generally been considered perfectly definite if the tests are made at standard temperature and voltage is kept on the potential coil long enough for it to reach a steady state. It was found, however, in this test that the heating effect of the series coils is by no means negligible. Evidently then the load curve obtained for a watthour meter depends to an appreciable extent on the time which is allowed to elapse after putting each load on the meter. In this discussion the electrodynamic load curve will be defined as one taken in so short a time that the heating effect of the series coils is negligible. In practice this can be very

¹ Exception was made for the Sangamo meter, as the smallest direct-current meter manufactured by the Sangamo Electric Co. is of 10 amperes capacity.

² Letters are used as a matter of convenience to designate the different manufacturers as follows: A, Columbia Meter Co.; B, Duncan Electric Manufacturing Co.; C, General Electric Co.; D, Sangamo Electric Co.; E, Westinghouse Electric & Manufacturing Co.; F, Willis Electric Meter Co. All of these meters are of the commutator motor type excepting D, which is a mercury meter.

nearly realized by taking quick comparisons of the rate at a given load with the rate at some one load as a standard; or, in other words, by alternating the readings at the two loads. In an actual test of this kind usually five checks of the meter rate are taken, alternately at the standard load and the load to be compared. This makes three checks at the standard load and two at the other

load. This procedure is then repeated for another load and the standard load, and so on. The full lines in Fig. 1 are curves obtained in this way, corrected to make the 10 per cent and 100 per cent load points correct.

Where corrections of this sort were made it was done on the assumption that shifting the magnets moves the load curve parallel to itself, while adjusting the starting device changes the rate by an amount inversely proportional to the load. Some

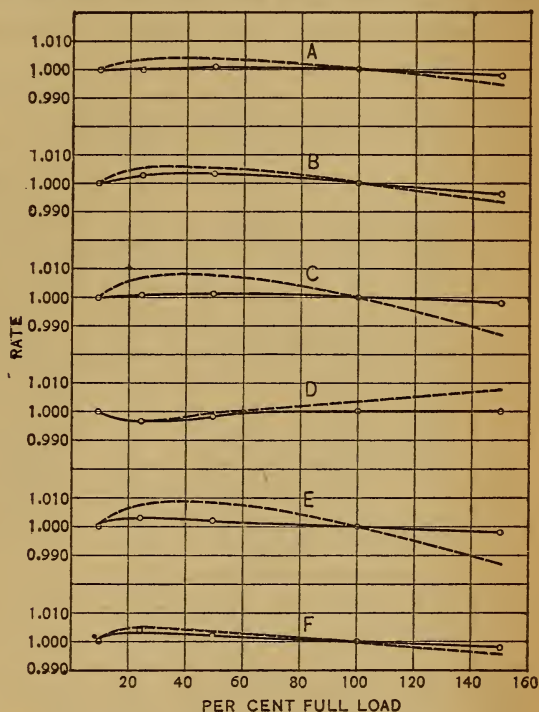


Fig. 1. ——— Electrodynamic load curves. The same with added effect of series coil heating

tests made on this point, of which the results are shown in Table 1 and discussed in paragraph 10 bear out the assumption closely except for meter D, and for this meter such correction was not attempted. The dotted lines in the same figure show the effect of the heating of the series coils in distorting the load curve. Here again the 10 and 100 per cent load points were adjusted to be correct. In Fig. 2 are given what may be termed the "ordinary" load curves; that is, no particular attention was paid to heating of the series coils, the points being taken

in as rapid succession as could be conveniently done, beginning at the light load. As before, corrections were applied to make the 10 per cent and 100 per cent load points correct, except for meter D. In section 16 will be found a further discussion of the load curves.

To determine the magnitude of the heating effect of the series element, a given load was held by means of a potentiometer after the potential circuit had come to a steady temperature and an

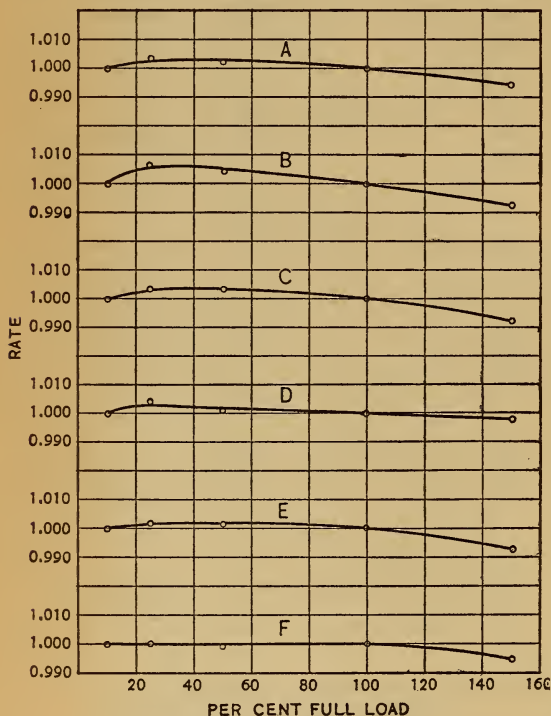


Fig. 2.—Ordinary load curves

sheet showing the change in rate of a meter with the lapse of time after closing suddenly on 150 per cent of full-load current. The chronograph drum made one revolution in 60 seconds; the vertical lines mark 5-second intervals. The curve is drawn through points indicated by a given number of revolutions of the meter disk. This curve would continue in the straight line AB if the rate of the meter had remained constant, but due to the heating of the series coil the rate decreased until the curve

automatic record of the revolutions of the meter was taken on a chronograph. From this record the rate of the meter at the beginning of the run could be obtained and the change in rate followed until no further change was noticeable. Such records were taken at several loads to determine the relation between the total change in rate and the load current. This change was found to be very nearly proportional to the square of the current. Fig. 3 is a typical chronograph

TABLE 1

Performance of Watthour Meters

Watthour meter	A	B	C	D	E	F
Rated full load R. P. M.....	33.0	36.7	45.8	27.5	45.8	55.0
Per cent difference in balance of current elements:						
No. 1.....	1.1	5.1	0.4		0.6	2.6
No. 2.....	3.2	1.5	1.6		0.0	3.2
No. 3.....	4.2	3.6	0.8		0.8	0.3
Per cent change from reversing stray field of 1 gauss at full load.....	2.2	2.2	2.3	0.0	2.6	2.7
Per cent change from removing covers:						
Heating ³	+0.3	+0.1	+0.6	0.0	+0.7	0.0
Magnetic.....	+0.2	0.0	-0.4	0.0	-0.3	0.0
Per cent change from reversing polarity:						
50 per cent load.....	0.8	0.6	0.5	2.9	0.9	0.5
100 per cent load.....	0.5	0.3	0.2	1.9	0.4	0.2
Polarity marked.....	Yes	Yes	No	No	Yes	Yes
Range of light-load adjustment, per cent.....	4.6	5.6	13.8	4.6	22.0	12.2
Range of full-load adjustment, per cent.....	86	155	68	35	92	65
Per cent change of full-load rate by maximum change of light-load adjustment.....	0.5	0.3	0.7	-1.5	1.5	1.1
Per cent change of light-load rate by maximum change of full-load adjustment.....	85	154	66	38	104	65
Per cent change from short circuit on 120 volts.....	2.5	2.9	1.1	2.3	1.9	-2.2
Maximum amperes on 120-volt short circuit.....	240	260	260	>500	270	340
Per cent change by short circuit on 240 volts.....	34.0	11.6	5.6	0.0	4.7	-3.4
Maximum amperes on 240-volt short circuit.....	400	400	430	730	440	560
Full-load back emf, in volts.....	0.13	0.13	0.19	0.0001	0.17	0.07
Starting current, in amperes:						
No. 1.....	0.06	0.02	0.04		0.09	0.08
No. 2.....	0.07	0.06	0.05	0.02	0.07	0.04
No. 3.....	0.07	(⁴)	0.03		0.08	0.11
Creeping voltage, in volts:						
No. 1.....	130	120	130	160	130	130
No. 2.....	130	130	130	260	130	130
No. 3.....	130	100	130	180	130	130

³ The plus sign indicates an increase in speed.⁴ Where no figure is given of the starting current it indicates creeping on voltage only.

coincided with the line CD. The dotted curves in Fig. 1, already referred to, were obtained from runs of this kind. The method of getting an automatic record of the rate of the meter on a chronograph sheet is described on page 174.

2. **Voltage Curves.**—In testing the meters for variation of rate with voltage they were run at full-load current and different voltages, each voltage being held a sufficient time for the resistance of the potential circuit to attain a constant value. The curves plotted in Fig. 4 show the results of the test. The rate of the meters is less at higher voltages; this effect is primarily due to the heating of the potential circuits, which have necessarily a large temperature coefficient. It should be noted that this performance would be slightly different at other loads, owing to the relatively larger effect of starting coil compensation at lighter loads.

TABLE 2
Temperature Coefficients (Per Cent per Degree C)

Watthour meter	A	B	C	D	E	F
Potential circuit.....	+0.17	+0.35	+0.41	+0.30	+0.41	+0.38
Disk.....	+ .39	+ .38	+ .39	+ .40	+ .37	+ .38
Magnets.....	- .03	- .01	- .01	- .01	- .02	- .03
Algebraic sum ⁶	+ .28	+ .05	.00	+ .12	.00	+ .06
Meter.....	+ .26	+ .10	+ .10	+ .12	+ .07	+ .11

⁶ In taking the sums, the coefficients of magnets were doubled and the sign reversed for both potential circuit and magnet coefficients.

3. **Temperature Coefficient.**—The temperature coefficient of a watt-hour meter depends upon the temperature coefficients of the potential circuit, the disk, and the drag magnets. Nearly complete compensation for temperature variations is ordinarily attained by making the temperature coefficients of the disk and potential circuit equal, as they affect the meter in opposite directions. This leaves outstanding the temperature coefficient of the magnets; this, however, is usually small. Table 2 gives the over-all coefficients of the meters and also the coefficients of the parts. It may be pointed out that the temperature coefficient of the meter depends directly on the algebraic sum of the coefficients of the disk and potential circuit but on twice the coefficient of the magnets. This is true because the rate of the meter is inversely proportional

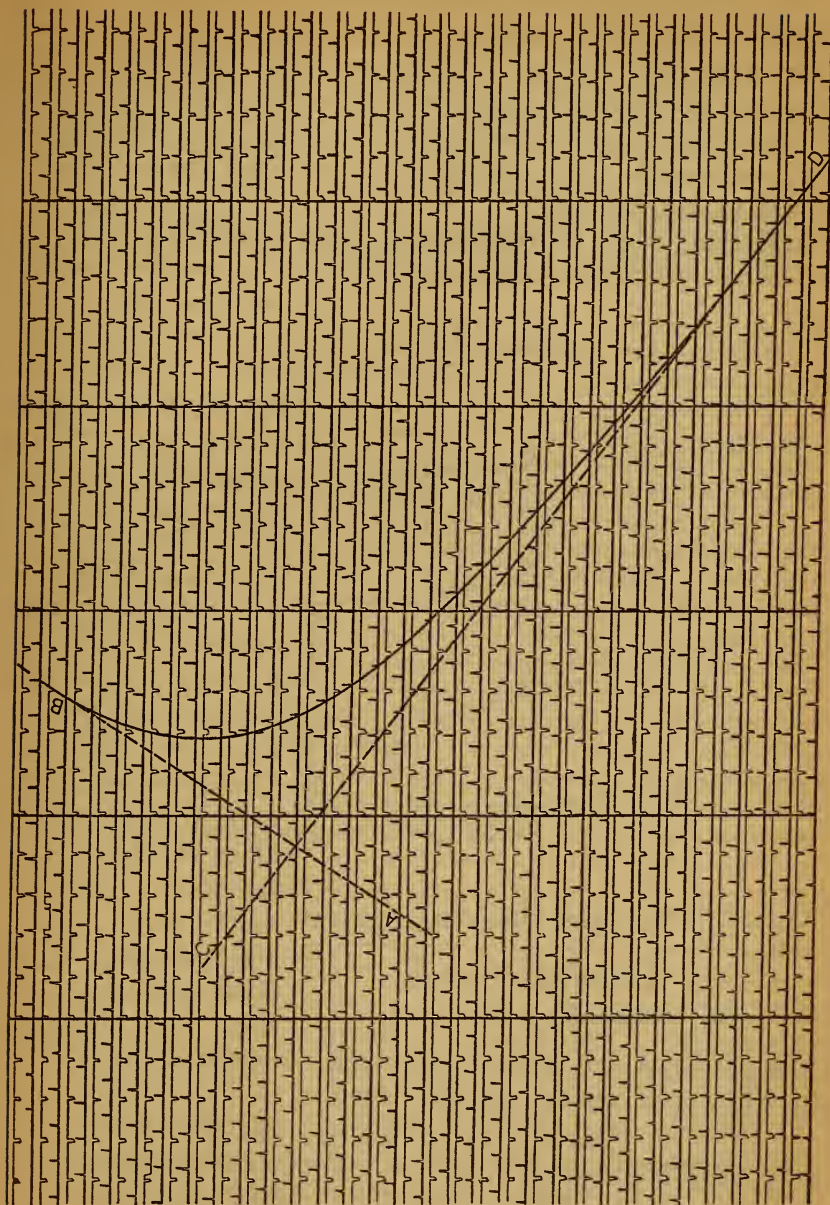


Fig. 3.—Typical chronograph record showing heating effect of series coils

to the square of the flux in the magnets. The check between algebraic sums and over-all coefficients of the meters is not very close for meters C and E. These meters show more effect due to heating of the series coil than the other meters and this effect may not have been altogether eliminated in getting the coefficients as watt-hour meters. In a summation of this kind small errors may happen to add together.

4. Equality of the Current Coils in Three-wire Meters.—In three-wire direct-current meters the potential circuit is generally connected

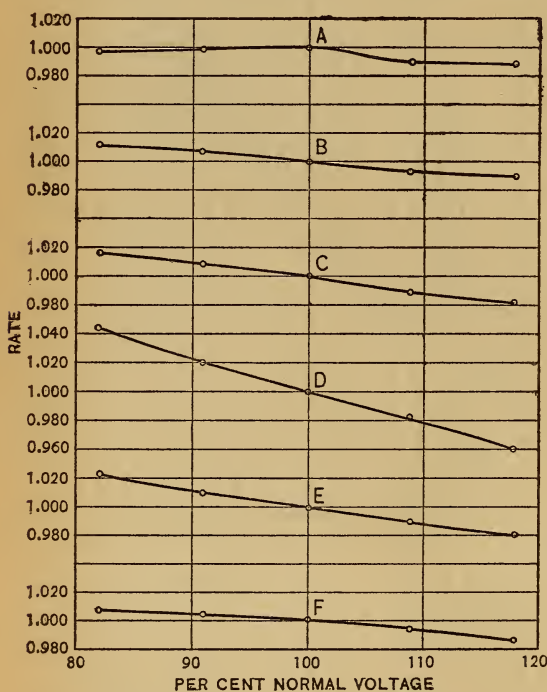


Fig. 4.—Variation of rate with voltage

between the neutral and one of the outside wires. With good voltage regulation the rate of the meter, with an unbalanced load, depends upon the equality of the two coils of the current element. To determine this equality of the coils, the rate of the meter was obtained by using each coil separately. The percentage difference in these rates is given in Table I, and is less than 2 per cent in most cases. Where the inequality of the coils amounts to 2 per cent the maximum error will be only 1 per cent, assuming the calibration to be correct when both coils are equally loaded.

5. Effect of Stray Field.—To determine this effect a field of one gauss was superimposed upon that of the current element in such a manner as to produce the greatest effect. This field is equal to that at a distance of 25 centimeters from a straight conductor carrying 125 amperes, and is about five times the horizontal component of the earth's field at Washington.

In Table 1 is given the percentage difference in rates due to reversing this field. This percentage change in rate should be equal to the percentage change in the meter field due to the stray field. This is seen to be very nearly true from the values of field strength given in Table 3, which are necessarily only approximate, as the fields are not uniform. The values given are for the average fields over the spaces in which the armatures rotate.

6. **Effect of Reversing Polarity.**—As meters are usually tested in position it is an open question whether the polarity should be

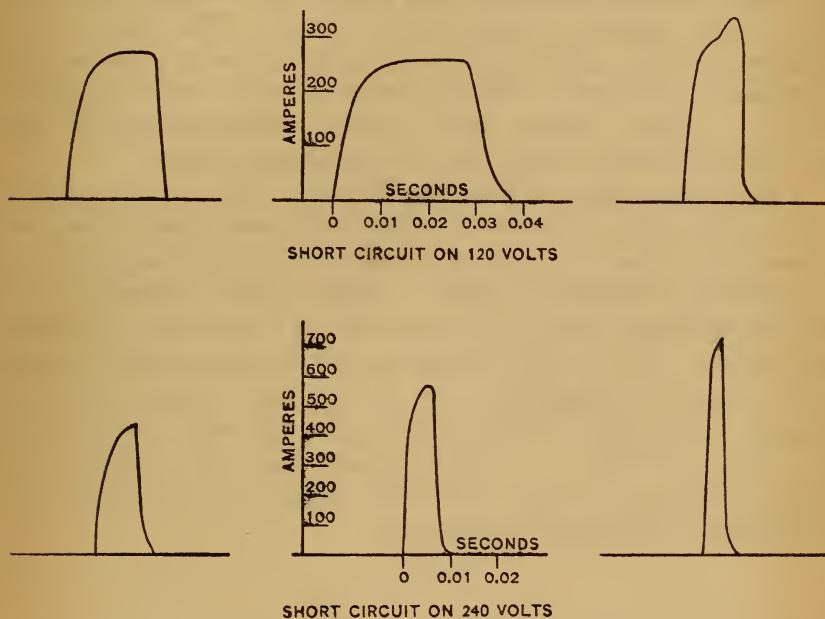


Fig. 5.—Oscillograms of short-circuit currents

marked. All the meters have the polarity marked except C and D. Owing to the thermocouple method of light-load adjustment on meter D it is clearly necessary to connect it in circuit with definite polarity, and it therefore appears desirable that its polarity should be marked. In Table 1 are given the effects of reversing polarity for both 50 per cent and full-load current.

7. **Effect of Covers.**—When a meter is tested in position it is usually necessary to remove the cover and often necessary to test it with the cover off. It is then a matter of importance to know what change in rate is caused by the removal of the cover. The

change in the rate may be due either to magnetic material in the cover or to temperature changes.

In Table 1 will be found the results of this test. Meters C and E, which show the largest amount of the magnetic effect, each have a small piece of sheet iron to hold the glass window in front of the dials.

8. Effect of Short Circuits.—As meters in service are always liable to be subjected to accidental short circuits, they should be constructed so that they will be as little affected as possible. To determine the magnitude of this effect the rate was obtained after ten times the full-load current had been passed through the meter. None of the meters was appreciably affected. The rate was also obtained after the meter was short-circuited through a fuse on 120 and then on 240 volts. The percentage change in rate is given in Table 1. Figure 5 shows some typical oscillograms which were taken in each case to determine the maximum current.

9. Starting Current and Creeping Voltage.—The current required to start the meters at normal voltage and the voltages required to make the meters creep when no current passes through the current element are given in Table 1. In most of the meters it is seen that the starting current is of the order of 1 per cent of the full-load current. Very small starting current is not necessarily a good feature, as it may indicate too small brush pressure for good contact. The values of brush pressure for one meter of each type are given in Table 4. As this pressure may vary widely even among meters of the same make, the values given are useful only in indicating the order of magnitude.

TABLE 3
Constants of the Coils

Watthour meter	A	B	C	D ⁶	E	F
Resistance of potential circuit in ohms:						
Multiplier.....	3080	1340				
Starting coil.....	230	12	1790		1710	
Armature.....	2300	1290	930		980	1360
Total.....	5610	2640	2720	5290	2690	4580
Potential circuit watts.....	2.2	4.6	4.4	9.2	4.5	2.6
Current element: Resistance, ohms.....	.26	.23	.23	.002	.21	.10
Watts loss, full load.....	6.5	5.7	5.7	0.2	5.2	2.5
Flux density full load, gaussses.....	70	85	75		70	85

⁶ Rated voltage of meter D was 220; of the others, 110.

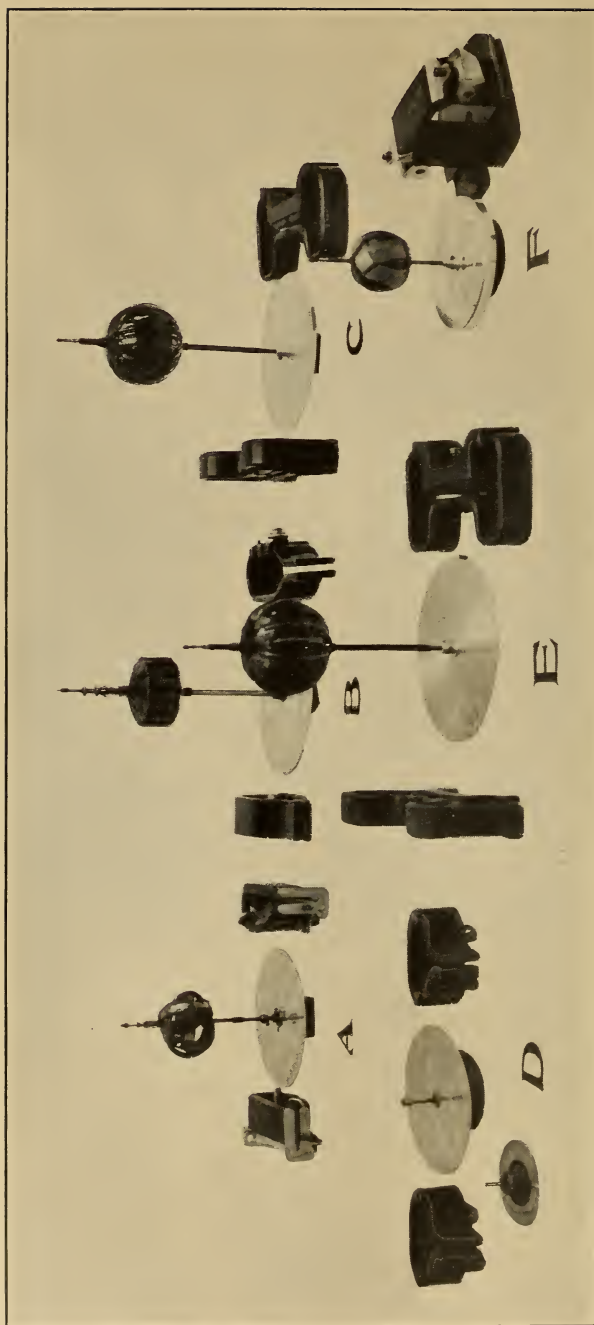


Fig. 6.—Moving elements and magnets

TABLE 4

Constants of Moving Elements

Watthour meter	A	B	C	D	E	F
Torque in cm-g.....	7.47	14.31	16.69	3.92	14.92	2.85
Weight in g.....	98.4	156.2	101.8	7.2	96.1	97.4
Ratio of torque to weight.....	.076	.092	.164	.545	.155	.029
Diameter of commutator, cm.....	.265	.465	.240		.240	.195
Number of commutator segments.....	3	8	8		8	3
Thickness of disk, cm.....	.115	.150	.065	.090	.065	.115
Diameter of disk, cm.....	11.40	13.35	12.66	10.18	12.70	8.54
Voltage drop across armature with 110 volts on potential circuit.....	45.1	53.8	37.6		40.1	32.7
Brush pressure, g.....	.24	.57	1.5		1.8	.34

TABLE 5

Constants of Magnets

[The dimensions are given in centimeters and the flux densities in gaussses]

Watthour meter	A	B	C	D	E	F
Number of magnets.....	2	2	4	2	4	1
Steel:						
Length, l_m	19.4	26.2	23.2	18.1	24.0	13.3
Cross section, q_m	2.2	2.0	1.4	2.2	1.4	2.5
Air gap:						
Length, l_a	.28	.36	.24	.29	.25	Adjustable
Cross section, q_a	5.3	6.8	4.4	3.1	4.4	2.5
$K=l_m/q_m+l_a/q_a$	167	247	304	88	302	
Total flux.....	9700	14400	7300	8100	7300	? 1300

⁷ Taken with 2.5-mm air gap.

10. **Range of Adjustments and Their Mutual Effects.**—In Table 1 it is seen that the range of the full-load adjustment is large compared with that of the light load, also that full-load adjustment shifts the whole-load curve, while the light-load adjustment shifts each point by an amount very nearly inversely proportional to the load. In the case of meter D, an increase in the rate at light load due to the shift of light-load adjustment caused a decrease in the rate at full load. This peculiarity is due to the thermocouple method of light-load adjustment. This arrangement consists of a thermocouple, heated by the potential circuit coil, which sends an auxiliary current through the mercury chamber. In other words,

this circuit is a shunt circuit on the mercury chamber. At low values of the main current this circuit adds to the current through the mercury chamber, but when the IR drop of the main current reaches a value equal to the thermocouple emf the auxiliary current becomes zero and for still higher values of the main current some current is forced backward through the thermocouple, making a reduction in the driving torque.

A large range of light-load adjustment is convenient for the meter tester, but this has its disadvantages. Too great a range of light-load adjustment will permit the meter to be adjusted to be correct when there is excessive friction and the meter should be either repaired or removed.

III. DETAILS OF CONSTRUCTION

11. The Moving Elements.—The main constants of the moving elements are given in Table 4. Each value of torque given is the average of 10 values reduced to 25° C, obtained with the rotating part in 10 positions, one-tenth of a turn apart. These measurements were taken with full-load current and normal voltage of 110 or 220 volts. It may be mentioned that when a meter is rated for a range of voltages, the highest voltage is generally used in measuring the torque.

12. The Magnets.—In Table 5 the number and the dimensions of the magnets are given, together with their magnetic data, determined by Dr. C. W. Burrows and Mr. R. L. Sanford. The quantity, K , appearing in this table is a double ratio—the ratio of the length of the magnet to its cross section divided by the ratio of the length of the air gap to its cross section.

Heinrich and Bercovitz⁸ divide this ratio by 100 and call that the factor of safety against demagnetization. This is purely a geometrical quantity and of course does not take into consideration either the composition, the temper, or the flux density of the steel. It is of use, however, in estimating the probable permanence of a magnet and in pointing out radical defects of magnet design.

13. Miscellaneous Details.—Figs. 7 and 8 are photographs of all the types of meters with and without covers, while Fig. 6 is a photograph of the moving elements and magnets. Various details of construction in the meters deserve special mention.

⁸ Heinrich und Bercovitz: *Handbuch der Electrotechnik* [5th ed.], 2, p. 29. Circular 20, Bureau of Standards, p. 17.

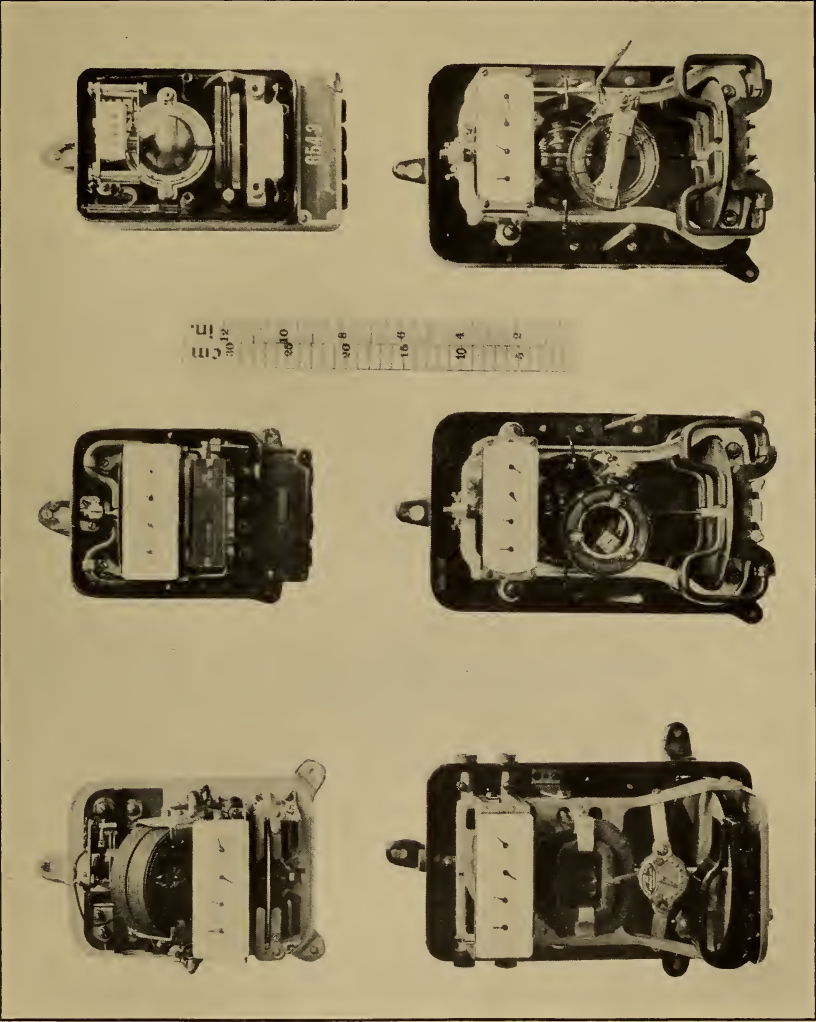


Fig. 8.—Meters with covers removed

The armatures of meters A, B, C, E, and F rotate from left to right, while that of D rotates from right to left. It is certainly unfortunate that all watthour meters, both direct current and alternating current, as well as ampere-hour meters, have not been designed to have the same direction of rotation. But since we do not have uniformity in this matter it seems but reasonable that the direction of rotation should be indicated by an arrow on every meter. Meters A, D, and F are so marked.

The spot on the disk should be large enough to be easily seen and should extend over the edge and along the bottom of the disk. While it is a mere detail, it would be a convenience in testing if this spot were located on the same radial line as the set screw of the disk. This would avoid the confusion which may arise at the higher speeds, and the spot could then be used at the beginning and the end of a run to secure greater accuracy. None of these meters were so arranged.

Meters A, D, and F have special arrangements for easily removing the dials for checking gear ratio; in meter F, however, this results in freeing the upper pivot. Meter F has a cyclometer dial with a device for equalizing the friction load on the armature. Meters B and F have special devices designed to render the covers more easily removed and replaced. Meters A, B, D, and F allow observation of the disk when the cover is on.

Meters A, D, and F have micrometer adjustments of the drag magnets for regulating the rate of the meter. Meters E and F use a steel ball and two jewels instead of a pivot and jewel bearing. Meters A, C, and E are provided with springs to take the weight of the moving element off the jewel when the meter is to be moved.

IV. DISCUSSION OF FRICTION LOSSES AND THEIR EFFECT UPON THE LOAD CURVES

14. **Separation of Friction Losses.**—There are four principal sources of friction in a watthour meter, the brushes, bearings, gearing, and windage. Few attempts have been made to determine the magnitude of these losses separately and to study their effect on meter performance. Schmiedel,⁹ however, has published a systematic investigation on this subject in which he gives a large number of

⁹ Schmiedel, *Verhandlung des Vereins zur Beförderung des Gewerbflusses*, Vols. 89-90, 1910-11. *The Electrical Review*, (London) 69, Dec. 22, 1911. *Science Abstracts*.

curves for the different losses both in alternating-current and direct-current meters.

To determine the frictional losses it is necessary to remove the drag magnets, as they produce most of the retarding torque, compared with which the friction torque is small. In physical measurements, other things being equal, the most direct method of measurement is to be preferred. Measurement of the power input for a given speed gives the torque directly in terms of the full-load torque, so from the standpoint of directness it is the best method. The difficulty with it arises from the flatness of the speed-torque curve over a considerable range of speeds, which causes the meter to run in a state of indifferent equilibrium. This method was

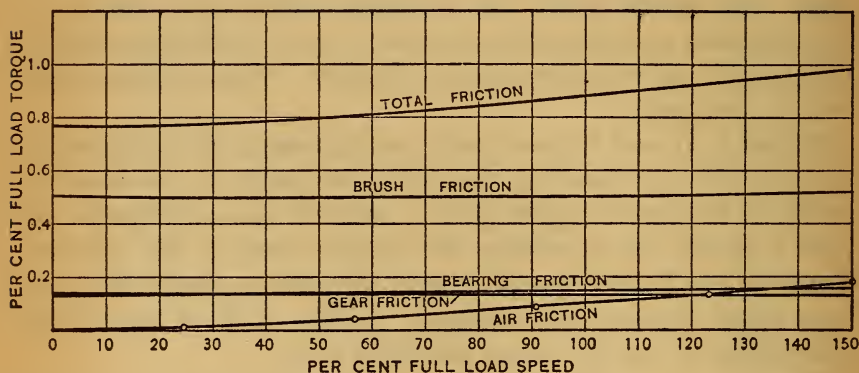


Fig. 9.—Friction torque, meter A

used only as a check except for meter D, which has large fluid friction due to the mercury.

In another method, which will be referred to as the decrement method, an automatic chronograph record was made of the revolutions of the moving element as its speed decreased from a given initial value to zero. From this record and the moment of inertia of the moving element the torque was calculated. In obtaining the chronograph record a transformer of small capacity giving about 3000 volts was arranged so that at each revolution of the moving element a spark was drawn from a rider on the disk to one high-tension terminal, the other terminal being grounded on the frame. Only the primary current of the transformer drawn at 120 volts was passed through the operating coil of the chronograph. An incandescent lamp in this circuit limited the current.

This arrangement gave automatic operation of the whole mechanism for getting the data of the revolution-time curves, while the torque introduced by the arc is inappreciable. The record of normal running of a meter shown in Fig. 3 was obtained by this

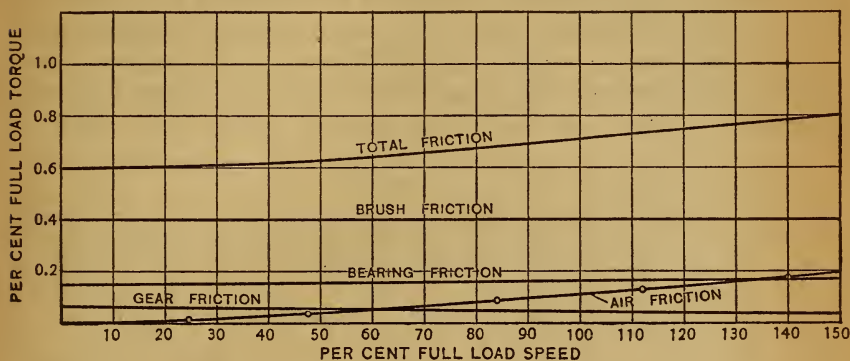


Fig. 10.—Friction torque, meter B

method. This method of timing meters has been developed independently and described in detail by Thompson.¹⁰ The speed-torque curves for the brush, bearing, and gear friction shown in Figs. 9 to 14, inclusive, were determined by this method. To get

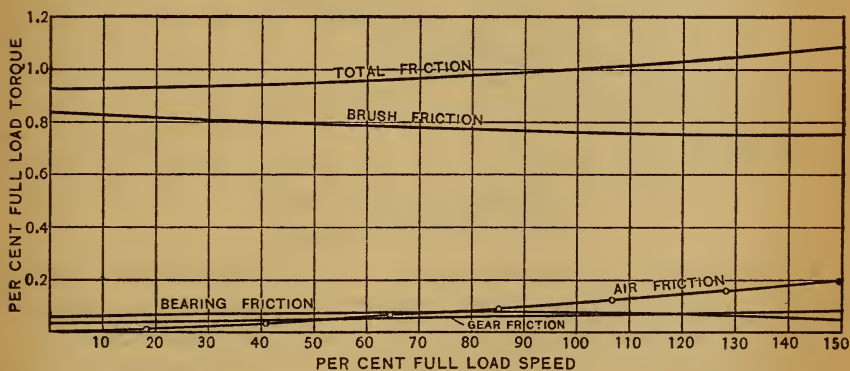


Fig. 11.—Friction torque, meter C

these values it was necessary to take chronograph records as described, first with brushes and gear train on, then with brushes off and gear on, and finally with both brushes and gear off. The last of these gives the windage and bearing friction, and the gear

¹⁰ Thompson: *Electrical World*, 61, p. 246; 1913.

and brush friction are obtained by taking differences. The air friction might be separated from the bearing friction in a similar manner by taking runs in a vacuum, but as the difficulties of this test are considerable, another method for getting the air friction was adopted. The drag of the air may be measured by suspending the moving element by a metal strip and rotating the case around it. As it was not convenient to use the cases of the meters, a box having approximately the same shape and giving about the same conditions as the case was used instead, except for meter B,

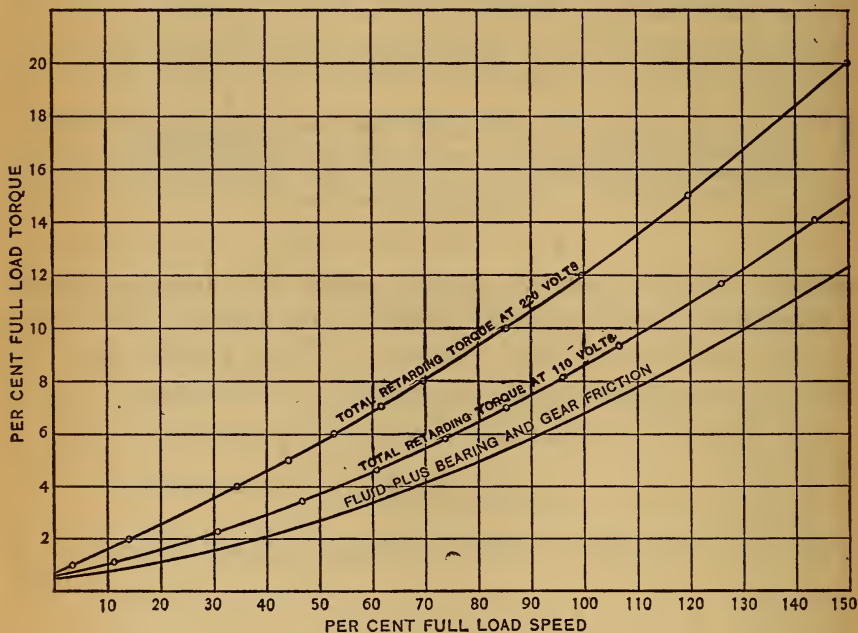


Fig. 12.—Friction torque, meter D

where both case and box were tried. As the results agreed closely for this meter, the box arrangement was deemed a sufficient approximation for the other meters. With the case revolving at a given speed the angle of torsion of the moving element was read off by means of a mirror with telescope and scale. The torque of the suspension strip per unit angle of twist was determined by obtaining the period of vibration with a disk of known moment of inertia replacing the watthour meter moving element.¹¹

¹¹ The torque in centimeter-dynes per radian ($57^{\circ}.3$) of twist is given by the equation

$$Q = \frac{4\pi^2 K}{T^2}$$

where K = moment of inertia in cgs units and T = period of vibration in seconds.

In Fig. 15 are given curves obtained by the decrement method with meter C to determine the retarding effect of the current in

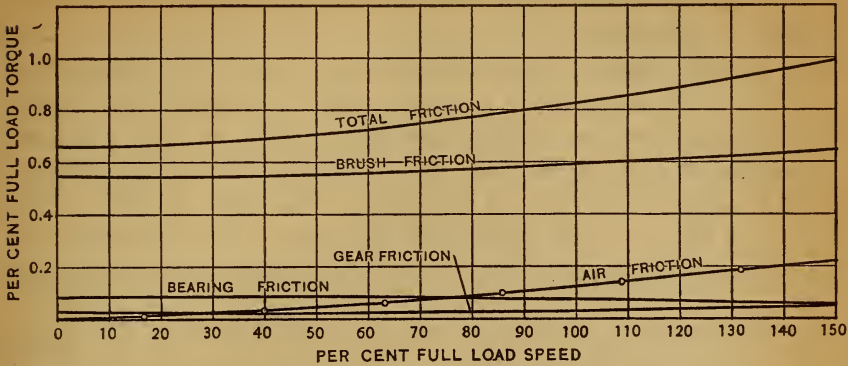


Fig. 13.—Friction torque, meter E

the current element. From these curves, which were taken with both brushes and gear off, it is seen that this retarding torque due to the current is comparable in magnitude with the bearing-

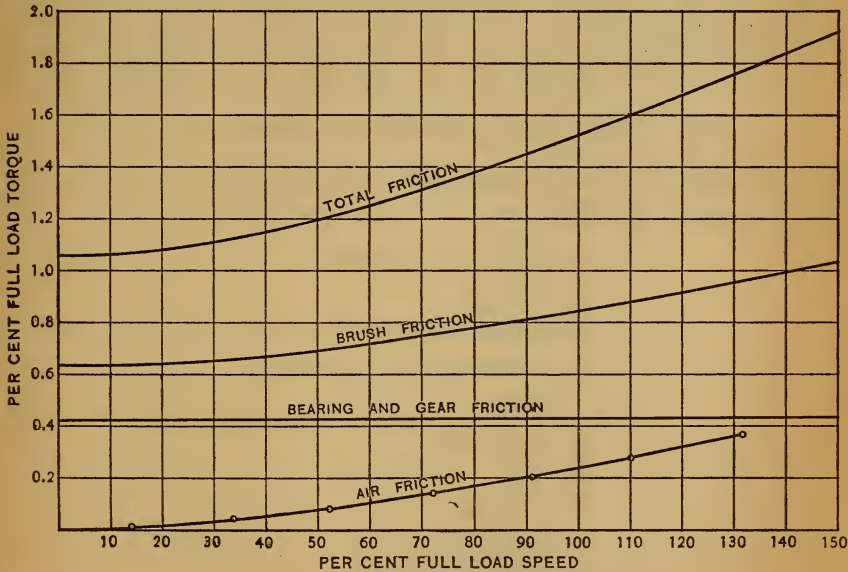


Fig. 14.—Friction torque, meter F

friction torque. After replacing the steel shaft by an aluminum one this effect was found to have practically disappeared. Meter A

showed a similar effect, though it has a brass shaft, but in this instance it appeared to be due to the drag action of the flux from the series coil on the disk itself, which is close to the current element in this meter.

To get the value of the friction torque from the chronograph records it was necessary to analyze the revolution-time curves, some typical examples of which are given in Fig. 16. By changing the algebraic sign of the time the revolution-time curve may be plotted with the instant of stopping as the zero of time. The curves of Fig. 17 were plotted in this way. The advantage of plotting the curves backwards is that the mathematical treatment

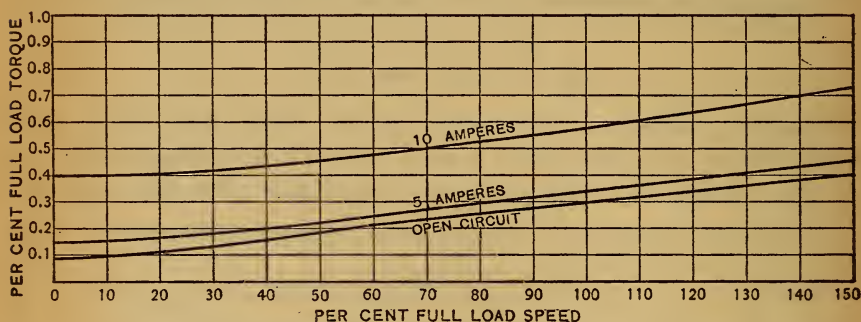


Fig. 15.—Friction torque, showing effect of current in series coils

of the problem is much simplified. The following set of symbols will be used in this discussion:

K = moment of inertia.

Q = torque.

θ = angle of rotation.

N = number of revolutions.

t = time.

R = resistance.

ϕ = flux.

E = voltage.

I = current.

ω = angular velocity.

a , b , c , and d = constants.

Considering the curve to be plotted as if the speed were increasing instead of decreasing, and assuming the friction torque to be constant, we may write the differential equation of motion:

$$K \frac{d^2\theta}{dt^2} = Q \quad (1)$$

$$K \frac{d\theta}{dt} = Qt + c_1$$

Integrating:

$$K\theta = \frac{1}{2}Qt^2 + c_1t + c_2$$

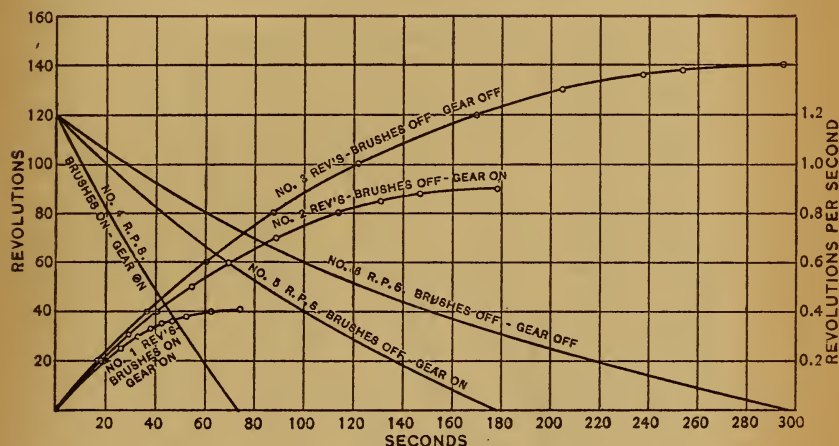


Fig. 16.—Nos. 1, 2, 3, revolution-time curves; Nos. 4, 5, 6, speed-time curves corresponding

When $t = 0$, $\theta = 0$ and $\frac{d\theta}{dt} = 0$,

since the meter is supposed to start from rest.

$$\therefore c_1 = c_2 = 0$$

We have then as the algebraic equation of motion,

$$K\theta = \frac{1}{2}Qt^2$$

$$\text{or} \quad \theta = \frac{1}{2} \frac{Q}{K} t^2 = ct^2 \quad (2)$$

This is the equation of a parabola, but it was found that it does not fit the retardation curves. Various assumptions might be made in writing the differential equation of motion instead of

assuming the friction torque to be constant, as was done above. But as it is not known just what assumptions to make, it appears better to assume N as a function of t in the first place, guided by the result in the simple case where the friction torque is assumed to be a constant. The simplest assumption to make is a power series:

$$N = a_0 + a_1t + a_2t^2 + a_3t^3 + \dots \quad (3)$$

The curves as actually taken are for retarded motion; but if a curve were to be taken with accelerated motion—as, for instance, by putting on a driving torque varying in such a way that it would

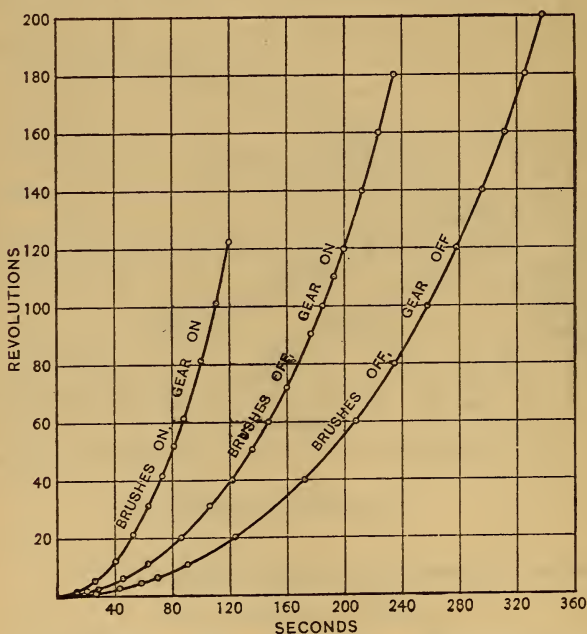


Fig. 17—Revolution-time curves plotted backwards

be at all times twice the retarding torque—there appears to be no reason for supposing that the shape of the revolution-time curve would be different. That is, N would have the same absolute value for the same absolute value of t positive or negative. In other words, N is a function of the even powers of t as shown in Fig. 18.

This eliminates odd powers of t . With these assumptions the equation may be written

$$N = at^2 + bt^4 \quad (4)$$

neglecting all powers of t above the fourth. This equation was found to fit the revolution-time curves of all the electrodynamic meters with close approximation.

To get the values of a and b for any given curve, it is necessary to solve two simultaneous equations:

$$\begin{aligned} N_1 &= at_1^2 + bt_1^4 \\ N_2 &= at_2^2 + bt_2^4 \end{aligned} \quad (5)$$

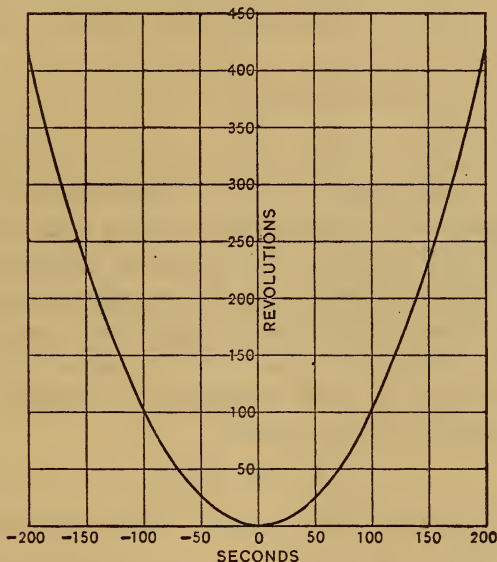


Fig. 18.—Revolution-time curve. Plot of equation $N = at^2 + bt^4$ where $a = 10^{-2}$, $b = 10^{-8}$

$$a = \frac{N_1 t_2^4 - N_2 t_1^4}{t_1^2 t_2^4 - t_1^4 t_2^2}$$

Solving:

$$b = \frac{N_2 t_1^2 - N_1 t_2^2}{t_1^2 t_2^4 - t_1^4 t_2^2} \quad (6)$$

Before making the solution as above, an approximate solution was first made for very small values of the time, where the fourth-power term is negligible in comparison with the square term. This

gives the fraction of a turn at the beginning of the run and the time corresponding to it. These quantities are usually determined from the first three whole turns of the speed-time curve as here considered, actually the last three whole turns of the meter before stopping.

Assume:

$$\begin{aligned} N_1 + x &= a(t_1 + y)^2 \\ N_2 + x &= a(t_2 + y)^2 \\ N_3 + x &= a(t_3 + y)^2 \end{aligned} \quad (7)$$

Where x is the last fraction of a turn and y the time taken for it.

Solving:

$$\begin{aligned} y &= \frac{(N_3 - N_2)(t_2^2 - t_1^2) - (N_2 - N_1)(t_3^2 - t_2^2)}{2[(N_2 - N_1)(t_3 - t_2) - (N_3 - N_2)(t_2 - t_1)]} \\ a &= \frac{N_2 - N_1}{t_2^2 - t_1^2 + 2y(t_2 - t_1)} \\ x &= a(t_1 + y)^2 - N_1 \end{aligned} \quad (8)$$

In some cases the values of x and y were measured; that is, the disk was watched as it was stopping and a punch made on the chronograph sheet by hand at the instant of stopping. When the disk was accurately leveled to avoid the effects of unbalancing, the measured values checked closely with those computed.

Having these correction terms for the turns and time, respectively, a solution can be made according to equations (5) and (6), giving the constants in the equation:

$$N = at^2 + bt^4.$$

The first derivative of this equation gives the speed in revolutions per second:

$$\frac{dN}{dt} = 2at + 4bt^3 \quad (9)$$

The second derivative multiplied by 2π gives the angular acceleration in radians per second per second, or the torque is given by the equation:

$$\begin{aligned} Q &= K \frac{d^2\theta}{dt^2} = 2\pi K \frac{d^2N}{dt^2} \\ &= 2\pi K(2a + 12bt^2) \quad \text{cm-dynes} \\ &= \frac{2\pi K}{981}(2a + 12bt^2) \quad \text{cm-g} \end{aligned} \quad (10)$$

Equations (9) and (10) give the necessary data for plotting the speed-torque curves.

Owing to the relatively large amount of fluid friction of the mercury meter D, the method of getting the friction torque from speed-time curves was not very successful. The rotating element, under this viscous damping only, will stop in 6 or 7 revolutions after being set in motion at twice full-load speed. The friction torque of this meter was obtained by running it as a watt-hour meter, but with magnets removed. Two speed-torque curves were taken in this way, one at 220 volts and the other at 110 volts. For the same value of ϕI —that is, for the same torque—there is a marked difference in speed at the two voltages. This is due to the difference in the damping caused by the flux from the potential circuit winding, which cuts the driving disk in the mercury chamber. From these two curves the viscous damping plus bearing and gear friction was deduced. This was done on the assumption that the magnetic drag is proportional to the square of the flux. The ratio of the flux at 110 to that at 220 volts was obtained by measuring the torque, first at 10 amperes 220 volts and then with 20 amperes 110 volts. From these values the amount of magnetic drag at 110 volts was computed for each point and a subtraction made giving the curve of total friction. (See Fig. 12.)

TABLE 6
Components of Torque

Torque	Unit	A	B	C	D	E	F
Brush friction.....	cm-g.....	0.038	0.057	0.127		0.090	0.024
	Per cent full load.....	.50	.40	.76		.60	.84
Gear friction.....	cm-g.....	.010	.006	.010		.004	
	Per cent full load.....	.13	.04	.06		.03	
Bearing friction.....	cm-g.....	.011	.023	.013		.012	.013
	Per cent full load.....	.15	.16	.08		.08	.44
Air friction.....	cm-g.....	.007	.016	.018		.019	.007
	Per cent full load.....	.10	.11	.11		.13	.44
Total friction.....	cm-g.....	.066	.102	.168	0.216	.125	.044
	Per cent full load.....	.88	.71	1.01	¹² 5.50	.84	1.52
Drag magnet.....	cm-g.....	7.40	14.21	16.52	3.70	14.80	2.81
	Per cent full load.....	99.12	99.29	98.99	94.50	99.16	98.48
Full load.....	cm-g.....	7.47	14.31	16.69	3.92	14.92	2.85

¹² Mostly liquid friction torque which partly replaces the retarding torque of the drag magnets.

The curves of Figs. 9 to 14 show the results of the work in separating the friction torques for one meter of each type. While the experimental difficulties are considerable, it is believed that the curves not only give the correct order of magnitude but that they are fair approximations to the actual quantities. As the friction forces, air friction excepted, vary from time to time, very accurate measurements would be of little value even if attainable. The general results are in good agreement with those of Schmiedel. From the curves given it appears that the air friction is probably responsible for nearly all the curvature in the curves for total friction torque. In the only curves given by Schmiedel for air friction torque alone, he shows them as straight lines; but it is possible that he considered the accuracy of determination of those particular curves was not sufficient to give the departure from a straight line. The friction torques due to the brushes, the gearing, and the bearings are constant within experimental error.

It should be stated that the brush friction, which in each instance constitutes more than half the total, does not depend on the type of meter alone, but depends in large measure upon the adjustment of brush pressure for each individual meter. Hence the data as given here on single meters can not be used to compare the different types.

15. Discussion of Load Curves.—The common explanation of the shape of the load curve of the commutator type of direct-current motor meters is that the rise in the curve between light load and full load is due to the increase in friction at lower speeds and that the droop at overload is due to the back electromotive force in the armature. Measurements of the back electromotive force show it to be inadequate to explain this droop, as may be seen by reference to table 1, which shows the largest value to be only 0.19 volt.

The explanation mentioned is not only superficial but is actually misleading, as it seems to demand that the curve of total friction should pass through a minimum, which will be shown to be unnecessary. There is some indication that at low speeds the curve of total friction torque passes through a slight minimum which the curves as drawn do not show. If a meter with the magnets

removed is driven as a watthour meter, it will not run steadily below 15 or 20 per cent of full-load speed. In other words, this is a region of instability of running. This would be a natural result if the friction torque curve has a slight minimum at low speeds, since for stable running it is evidently necessary that the retarding force should increase with the speed—a condition always fulfilled when the magnets are on, at least within the range of working speeds.

The electrodynamic load curve will be considered first; that is, the curve as it would be were there no heating effects from the current in the series coils. Let AB in Fig. 19 represent the total

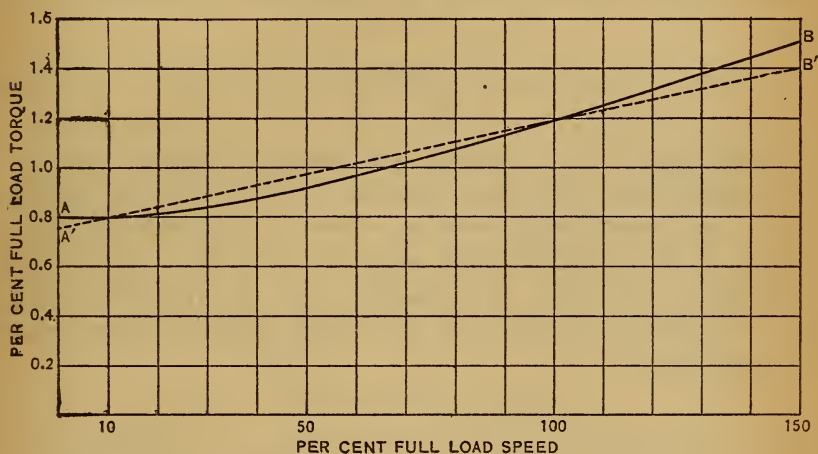


Fig. 19.

friction torque curve. Draw the straight line A' B' through the values of the torque at 10 per cent and 100 per cent speed. If the starting torque were made to compensate for the friction at the 10 per cent point and if the friction increased directly as the speed, or, in other words, followed a linear law such as indicated by the straight line A' B', the speed of the meter would be proportional to the driving torque. That is, the friction torque could be added in as part of the retarding torque and would follow the same law as the retarding torque due to the magnets. As the retarding friction torque actually follows a curve concave upward, its value is too great below 10 per cent load, too small

for loads between 10 per cent and 100 per cent, and too great above 100 per cent load. The meter as a consequence tends to run slow at less than 10 per cent load, fast between 10 per cent and full load, and slow at overloads. All this depends on the assumption that the meter is adjusted to be correct at the 10 per cent and full load points, though the general kind of deformation of the load curve would be the same whatever the adjustment. Applying this theory to the actual friction curves of the meters it is found that, taken together with the back electromotive force, it practically accounts for the shape of the electrodynamic load curve.

The driving torque Q_d may be analytically expressed in the form:

$$Q_d = C_d + DI \left(\frac{E - a\omega}{R} \right) = C_d + D_1 I - D_2 I \omega \quad (11)$$

where the constant C_d represents the starting torque of the light-load adjustment, I the load current, E the impressed voltage, $a\omega$ the back electromotive force, which is proportional to the speed ω , R the potential circuit resistance, and D , D_1 and D_2 proportionality constants. In a similar way the retarding torque may be expressed in the form of a series, omitting terms greater than the square:

$$Q_r = C_r + D_3 \omega + D_4 \omega^2 \quad (12)$$

where C_r is the initial value of the friction torque, $D_3 \omega$ the retarding torque of the drag magnets, and $D_4 \omega^2$ represents the change in friction torque with speed.

For the condition of running:

$$Q_d = Q_r$$

Subtracting (12) from (11) gives:

$$C_d - C_r + D_1 I - D_2 I \omega - D_3 \omega - D_4 \omega^2 = 0$$

Solving:

$$\omega = \frac{1}{D_3} (C_d - C_r + D_1 I - D_2 I \omega - D_4 \omega^2) \quad (13)$$

Since $D_2 I \omega$ and $D_4 \omega^2$ are only small correction terms they may be replaced by their approximate values $D'_2 I^2$ and $D'_4 I^2$, respectively, assuming the speed to be proportional to the current.

Substituting these values in (13) gives:

$$\omega = \frac{I}{D_3} [C_d - C_r + D_1 I - (D'_2 + D'_4) I^2] \quad (14)$$

If the meter does not creep at no-load:

$$C_r > C_d$$

Equation (14) may then be rewritten:

$$\omega = -C_0 + C_1 I - C_2 I^2$$

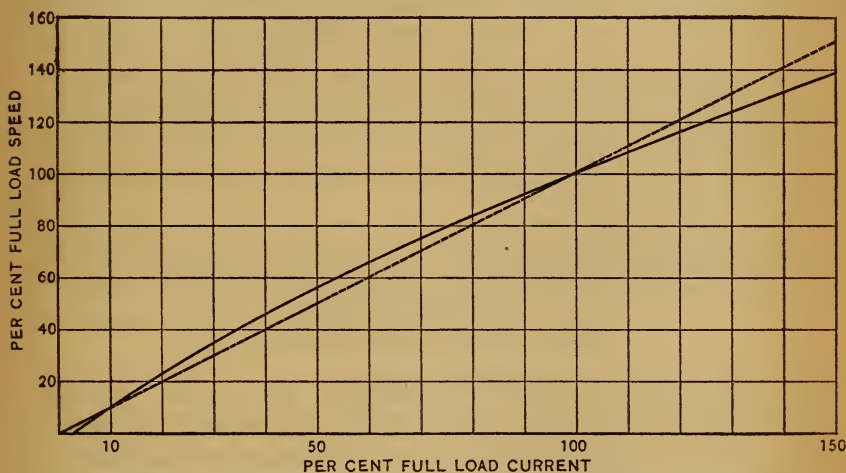


Fig. 20.—Speed-current curve, showing variation from a straight line

The negative sign is chosen for C_0 to indicate that under the assumed conditions the meter will not start until some small value of the line current is attained. The term involving I^2 is very small, amounting to only three or four thousandths of the term involving I even at the largest values of the current. Figure 20 represents graphically the variation of angular velocity with current in the series coils according to equation (15). A straight line may be drawn through the zero cutting the curve in two points and so the ratio of the velocity to the current is the same for these two points.

The significance of this is that a meter may be adjusted to be correct at two loads only. The departure of the curve from the straight line, as drawn, is greatly exaggerated.

The dotted lines of Fig. 1 show the effect of heating of the series coils upon the load curve.¹³

The reason for the unavoidable increase in the rise of these curves between 10 per cent and full load can now be understood. In an ideal meter where there would be no heating, the load curve would be the electrodynamic curve *AB* of Fig. 21. In the actual

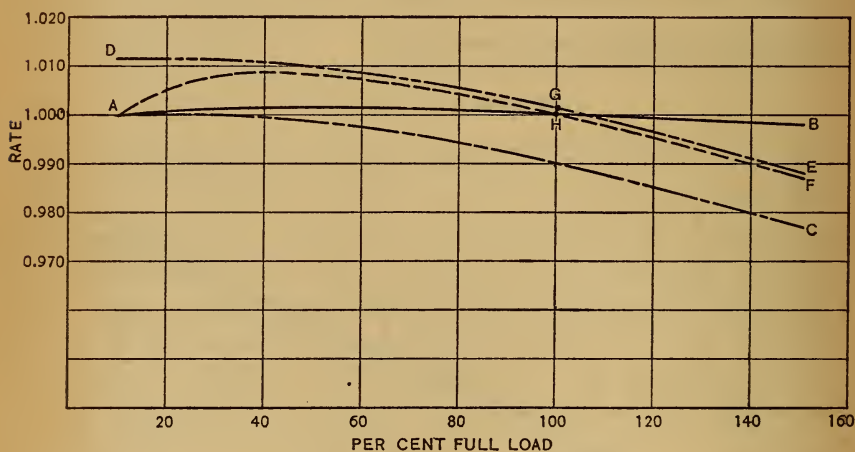


Fig. 21.—Effect of heating of series coils

meter the current coils have resistance, and as the heating is proportional to the square of the current, the meter rate is reduced much more at the higher loads than at lower, so that the curve *AB* changes into *AC*. The curve *AC* may now be shifted parallel to itself by the full-load adjustment until it takes the position *DGE*, such that when the 10 per cent load point is made correct, the full-load point will be correct. The point *D* is made to coincide with *A* by changing the friction compensation. The curve will now have the position *AHF*, showing the meter to be

¹³ The heat developed in the series coils causes the resistance of both armature and disk to be increased. The increase of armature resistance causes a decrease in driving torque with a corresponding decrease in the rate of the meter. The increase of the resistance of the disk causes a decrease in retarding torque, but as the disk is located at a greater distance from the source of heat than the armature the changes in these torques do not balance.

correct at 10 per cent and full load but running fast at intermediate loads where the effect of light-load compensation and heating are both small.

An examination of the various load curves in Figs. 1 and 2 shows that the heating effect of the series coils is the chief factor in determining the shape of the load curves, as the deformations due to it are greater than those due to the friction and back electromotive force.

We are indebted to Dr. P. G. Agnew for many valuable suggestions in connection with this work.

V. SUMMARY

1. A large amount of numerical data is given in regard to the performance and construction of six types of American meters. (See Tables 1 to 5, pp. 165, 166, 170, 171.)

2. The total friction torque is separated into four parts—brush, gearing, bearing, and windage friction—and curves are plotted of these components of torque against speed. A description is given of the method used in obtaining these curves. (See Table 6, p. 183.)

3. The load curve is analyzed and its shape is shown to depend upon the heating of the series element, the variation in the friction torque, and the back electromotive force.

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